Communicate Your Understanding

C1 Describe the steps needed to complete the square for each relation.

a) $y = x^2 + 10x + 15$

b) $y = -2x^2 - 4x - 5$

C2 Identify the vertex of each relation in question C1. How do you know whether it is a maximum or a minimum point?

C3 Explain how to graph an equation of the form $y = a(x - h)^2 + k$.

Practise

For help with questions 1 to 6, see Example 1.

1. Use algebra tiles to rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = x^2 + 2x + 5$

b) $y = x^2 + 4x + 7$

c) $y = x^2 + 6x + 3$

2. Determine the value of c that makes each expression a perfect square.

a) $x^2 + 6x + c$ **b)** $x^2 + 14x + c$

c) $x^2 - 12x + c$ **d)** $x^2 - 10x + c$

e) $x^2 + 2x + c$ **f)** $x^2 - 80x + c$

3. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = x^2 + 6x - 1$

b) $y = x^2 + 2x + 7$

c) $y = x^2 + 10x + 20$

d) $y = x^2 + 2x - 1$

e) $y = x^2 - 6x - 4$

f) $y = x^2 - 8x - 2$

g) $y = x^2 - 12x + 8$

4. Find the vertex of each quadratic relation by completing the square.

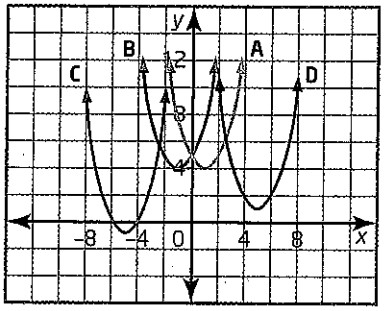
a) $y = x^2 + 6x + 2$

b) $y = x^2 + 12x + 30$

c) $y = x^2 - 8x + 13$

d) $y = x^2 - 6x + 17$

5. Match each graph with the appropriate equation.



a) $y = (x - 5)^2 + 1$

b) $y = (x - 1)^2 + 4$

c) $y = (x + 1)^2 + 4$

d) $y = (x + 5)^2 - 1$

6. Find the vertex of each parabola. Sketch the graph, labelling the vertex, the axis of symmetry, and two other points.

a) $y = x^2 + 10x + 20$

b) $y = x^2 - 16x + 60$

For help with questions 7 to 9, see Example 2.

7. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = -x^2 + 80x - 100$

b) $y = -x^2 - 6x + 4$

c) $y = 3x^2 + 90x + 50$

d) $y = 2x^2 - 16x + 15$

e) $y = -7x^2 + 14x - 3$

8. Find the maximum or minimum point of each parabola by completing the square.

a) $y = -x^2 - 10x - 9$

b) $y = -x^2 + 14x - 50$

c) $y = 2x^2 + 120x + 75$

d) $y = 3x^2 - 24x + 10$

e) $y = -5x^2 - 200x - 120$

9. Use Technology Use a graphing calculator to find the maximum or minimum point of each parabola, rounded to the nearest tenth.

a) $y = x^2 + 6x - 1$

b) $y = 0.2x^2 - 1.5x + 6.3$

c) $y = -1.6x^2 + 4.3x - 5.2$

d) $y = \frac{1}{2}x^2 - \frac{1}{8}x + \frac{1}{2}$

e) $y = -57x^2 + 91x - 13$

f) $y = 144x^2 + 25x + 14$

For help with questions 10 and 11, see Example 3.

10. Find the vertex of each parabola. Sketch the graph, and label the vertex and two other points.

a) $y = -x^2 - 2x - 6$

b) $y = 4x^2 + 24x + 41$

c) $y = 5x^2 - 30x + 41$

d) $y = -3x^2 + 12x - 13$

e) $y = 2x^2 + 8x + 3$

11. Find the vertex of each parabola. Sketch the graph, and label the vertex and two other points.

a) $y = -2x^2 - 3x + 7$

b) $y = 3x^2 - 9x + 11$

c) $y = -x^2 + 8x - 10$

d) $y = 4x^2 - 16x + 11$

e) $y = -5x^2 - 30x - 48$

Connect and Apply

12. The path of a ball is modelled by the equation $y = -x^2 + 4x + 1$, where x is the horizontal distance, in metres, travelled and y is the height, in metres, of the ball above the ground. What is the maximum height of the ball, and at what horizontal distance does it occur?

13. Use Technology A football is kicked at an angle of 30° to the ground, at an initial speed of 20 m/s, from a height of 1 m. Two quadratic relations can be used to model the height, in metres, above the ground:

With respect to time, t, in seconds, the height is given by $h = -4.9t^2 + 10t + 1$.

With respect to the horizontal distance, x, in metres, the height is given by $h = -0.0163x^2 + 0.5774x + 1$.

Use a graphing calculator to verify that the maximum height is the same with both models.

At what time and horizontal distance does the maximum height occur?

Did You Know?

Galileo was a mathematics professor at the University of Pisa, in Italy. At the beginning of the 17th century, he discovered the connection between quadratic equations and acceleration.

14. A diver dives from the 3-m board at a swimming pool. Her height, y, in metres, above the water in terms of her horizontal distance, x, in metres, from the end of the board is given by $y = -x^2 + 2x + 3$. What is the diver's maximum height?

15. The cost, in dollars, of operating a machine per day is given by the formula $C = 2t^2 - 84t + 1025$, where t is the time, in hours, the machine operates. What is the minimum cost of running the machine? For how many hours must the machine run to reach this minimum cost?